

Traffic Capacity of Large WDM Passive Optical Networks

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Abstract—As passive optical networks (PON) are increasingly deployed to provide high speed Internet access, it is important to understand their fundamental traffic capacity limits. The paper discusses performance models applicable to wavelength division multiplexing (WDM) EPONs and GPONs under the assumption that users access the fibre via optical network units equipped with tunable transmitters. The considered stochastic models are based on multiserver polling systems for which explicit analytical results are not known. A large system asymptotic, mean-field approximation is used to derive closed form solutions of these complex systems. Convergence of the mean field dynamics is proved in the case of a simple network configuration. Simulation results show that, for a realistic sized PON, the mean field approximation is accurate.

I. INTRODUCTION

Figure 1 illustrates the main components of a passive optical network (PON). Optical network units (ONUs) are situated in or close to user premises. An ONU might be dedicated to an individual user or, when fiber is terminated at the curb or at the building, it could concentrate the traffic of several users. The ONUs are controlled from an optical line termination (OLT) equipment that realizes the interface with the wide area network. Passive splitters are used to broadcast downstream optical signals to the ONUs and to merge upstream signals destined to the OLT.

To increase the capacity of a PON, it is proposed to employ wavelength division multiplexing (WDM) to multiply the capacity of the physical fibre. Typically the ONUs will not be equipped to use all wavelengths simultaneously. In this paper we assume an ONU has one or more transmitters that can be tuned to any wavelength. The spectrum is thus shared efficiently but the capacity of an ONU is limited by its quota of tunable transmitters¹.

The major PON design concern is the dynamic bandwidth allocation (DBA) algorithm implemented by the OLT to arbitrate use of upstream capacity and to avoid collisions. There are two PON standards, EPON from IEEE and GPON from ITU, that differ in the way they manage resource sharing. EPON relies on asynchronous transmission of Ethernet packets while GPON implements a synchronous 125 μ s frame enabling both circuit and packet switching. We develop generic models for packet based services that are applicable for both standards.

¹Equivalent capacity constraints would arise if the PON were to use one or more reflective semi-conductor optical amplifiers in the ONUs [12]

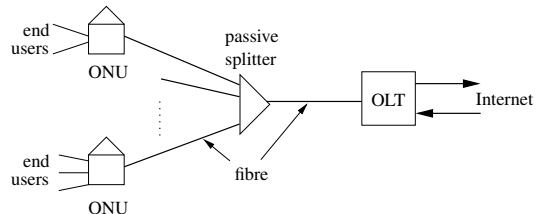


Fig. 1. PON components: ONUs, passive splitters and OLT

The focus of the paper is on traffic capacity, the limit on demand beyond which some ONU queues would always be saturated. This is arguably the most critical performance criterion for the PON whose very high bit rate ensures delays are tiny at normal loads and only become significant as demand attains the capacity limit (see [1], [13]).

We assume the OLT manages a single queue per user, receiving queue size *reports* from the ONUs and attributing corresponding *grants* for upstream transmission. A grant is the time interval during which a user may transmit. We maintain that such simple management at PON level (i.e., no accounting for intra-ONU class of service differentiation at the OLT) is highly desirable to limit complexity and generally meets user requirements: the OLT ensures users receive a fair share of bandwidth, the ONUs realize what service differentiation is needed within that share by choosing which packets to send to fulfill the allocated grants.

The objective of fair allocation naturally leads to a form of polling. For EPON we define a DBA that realizes a classical polling system with a particular interpretation for the switchover time between queues. The same model can be applied to one possible DBA implementation for GPON. A second considered GPON DBA, on the other hand, is more naturally modelled as a head of line processor sharing system. See below for more detail.

The underlying polling systems are complicated by the fact that ONU capacity is limited by the number of tunable transmitters they have, i.e., the number of servers that can be present simultaneously at an ONU. In particular, stability conditions of multiserver polling systems with limited-gated policies appear intractable in general.

Related work: There is an abundant literature on PONs and numerous DBA algorithms have been proposed. Two

recent surveys, by McGarry *et al.* [16] and by Zheng and Mouftah [23], present a comprehensive review for EPON. Skubic *et al.* [19] compare DBAs used in EPON with alternatives proposed for GPON. The DBA algorithm we propose is similar in principle to IPACT, proposed by Kramer *et al.* [10] for a regular EPON and extended by Kwong *et al.* for WDM [11]. More complex class of service aware DBAs for a WDM EPON are proposed in [5], [13]–[15], [17]. A paper by Gliwan *et al.* discusses DBA for a coarse WDM GPON [8]. Song *et al.* recognize that classical EPON DBAs can lead to lost capacity in long reach EPONs where propagation times are large and suggest the ONUs should initiate multiple interleaved cycles or threads [20]. We advocate an alternative DBA that avoids loss of capacity whatever the propagation times [1].

Most performance evaluations of DBA algorithms rely on simulation. A few authors have developed analytical models, generally using techniques developed for polling systems. Park *et al.* derived closed form formulas for the mean packet delay in a symmetric PON with identical ONUs and negligible propagation times under the assumption of Poisson traffic and gated service discipline [18]. The two-stage polling system identified in that paper has been further analyzed by van der Mei and Resing [22] when ONUs have heterogeneous load. To the best of our knowledge, polling models have not been developed explicitly for WDM PONs. The capacity of classical multiserver polling models was analysed by Fricker and Jaibi [7]. Borst and van der Mei introduced the notion of server limit in [3] but did not evaluate system capacity. Down [6] presents several capacity results for multiserver polling models with server limits but with unlimited service policies. Lastly, the study of head of line processor sharing systems by Brandt and Brandt is relevant but unfortunately provides no exact capacity results [4].

To overcome the complexity of the polling systems associated with these networks, the main contribution of the paper consists in the analysis of a large system limit where both the number of ONUs and the number of wavelengths grow indefinitely together. This scaling, also called *mean field limit*, provides explicit formulas and simple numerical algorithms to derive useful approximations for realistically sized PONs. It is based on the fact that, when the approximation holds, the ONUs become statistically independent in the limit, their interaction being represented through a deterministic equation. See [21] for a general presentation of these methods. Mean field limits have been used for some time now in queueing theory notably to study circuit switched networks, see [9] for example.

In Section II we outline our proposed DBAs for EPON and GPON. We then recall relevant multiserver polling system results and discuss the difficulty of an exact analysis of the WDM PON. The large system asymptotic is developed in Section IV where practically useful traffic capacity results are derived using the mean field assumption. Convergence is proved for a particular network configuration in Section V. The validity of this assumption is tested in Section VI and used to evaluate the impact of switch overhead on PON traffic

capacity. Section VII concludes the paper.

II. DYNAMIC BANDWIDTH ALLOCATION

The DBAs we consider for both EPON and GPON relate to a single, classless queue per user. The objectives are to ensure low latency by imposing per-visit grant limits and to provide users adequate throughput by controlling fairness.

A. EPON DBA

The EPON DBA is governed by the multi-point control protocol (MPCP) based on the exchange of REPORT and GATE messages between OLT and ONUs, as specified in IEEE 802.3ah. An extension to WDM is proposed by McGarry *et al.* [15]. ONUs report current contents of their user queues via REPORT messages while GATE messages are sent downstream by the OLT to allocate grants for each queue. In [1], we proposed a novel DBA where the OLT precisely times the sending of GATE messages and the upstream return of data packets and REPORTs to maximize traffic capacity. The timing is designed to compensate for differences in propagation times and to avoid wasting capacity when propagation times are large in long reach EPONs.

The EPON behaves then like a polling system. The ONUs are “visited” by each wavelength following a certain schedule. The visit enables the transmission of upstream data and a REPORT. The grants account for maximum per queue quotas. Each visit incurs a switch overhead time equal to the time needed to send the REPORT plus a physical layer guard time. The overhead is around $2\mu\text{s}$ for a 1Gb/s EPON [19]. The OLT is aware of all allocations and readily implements constraints on the number of simultaneous transmissions an ONU can sustain.

B. GPON DBA

The GPON standards define a particular framework for DBA that allows resource sharing between several communication services (see the G.984 series of ITU recommendations). We consider only the packet switched service. Ethernet packets are transmitted within GPON transmission convergence (GTC) layer frames of $125\mu\text{s}$ fixed duration. Downstream GTC frames are also used to convey grants from OLT to ONU users. Reports on queue contents are included in upstream GTC frames from ONU to OLT. Grants in a downstream GTC frame attribute transmission time slots in a precise upstream GTC frame occurring after a fixed time offset. The offset is calculated to allow all ONUs to fulfill their grants accounting for processing time and propagation delays.

It is possible to design a DBA in this framework that emulates a polling system. Wavelength capacity is allocated sequentially to ONUs for data transmission, or just for reporting if the queues are currently empty. Overhead is incurred on switching from one ONU to the next, as for the EPON. However, this overhead is rather small according to the GPON specification, accounting for only some 100ns [19]. An additional fixed overhead is incurred at the start of each frame.

The GPON DBA might alternatively consist in providing every ONU queue a share of every upstream GTC frame on one or several wavelengths. The OLT would calculate an allocation for each queue and a schedule of transmissions based on previously received reports and accounting for ONU transmission capacity. In this way, there is a fixed overhead per frame, accounting for reports and physical constraints, to be subtracted from the overall capacity. The remaining capacity would be shared between users according to some fairness policy.

III. MULTISERVER POLLING SYSTEMS

We introduce notation and discuss known polling system results that can be useful to analyse WDM PONs.

A. Notation and assumptions

In the following wavelengths are identified with servers circulating among ONUs. The polling system has N ONUs and L wavelengths. ONU i is equipped with t_i tunable transmitters and manages n_i independent user queues, i.e., t_i servers can be present at ONU i simultaneously. When the DBA emulates a polling system, the j^{th} queue of ONU i , queue (i, j) , has a maximum grant of d_{ij} seconds per visit by any wavelength. This mechanism is known as the limited-gated service policy. Finally, Δ_i is the switch overhead time associated with ONU i .

We make no assumptions about the traffic in the user queues except that the packet arrival process is stationary and has a well defined rate. The traffic intensity of queue (i, j) is ρ_{ij} expressed in units of the bit rate of one wavelength. We define $\rho_i = \sum_j \rho_{ij}$ and $\rho = \sum_i \rho_i$.

A wavelength is assumed to visit ONUs in some order, assumed different for each wavelength. For *periodic* polling, visits to ONUs occur according to a deterministic schedule. A wavelength that should visit an ONU with no free transmitter is assumed to skip directly to the next ONU in the schedule. Alternatively, for *random* polling, the next ONU to visit is chosen at random from the ONUs that currently have a free transmitter.

B. Limited grants, no server limits

In this section we assume $t_i = L$ for all i – an ONU can be served by an arbitrary number $\leq L$ of servers with a limited-gated policy. Polling is either periodic or random.

Proposition 1. *All queues in all ONUs are stable if and only if, for all i, j ,*

$$\rho + \frac{\rho_{ij}}{d_{ij}} \sum_{k=1}^N \Delta_k < L. \quad (1)$$

When not all queues satisfy (1), there is a subset \mathcal{S} whose members are saturated while, for $(i, j) \notin \mathcal{S}$, we have

$$\rho - \sum_{(l,k) \in \mathcal{S}} \rho_{lk} + \frac{\rho_{ij}}{d_{ij}} \left(\sum_{k=1}^N \Delta_k + \sum_{(l,k) \in \mathcal{S}} d_{lk} \right) < L. \quad (2)$$

Proof: Condition (1) derives from Theorem 3 in [7] or Theorem 2.4 in [6] with appropriate changes in notation. Relaxed conditions in (2) are deduced from the discussion in Section 4 of [7]. In fact, the results of [7] and [6] must be modified slightly since grant limits here apply to transmission time and not packets but this is straightforward. ■

We later require the following additional result.

Proposition 2. *When the system is stable, the mean time C between successive visits of wavelengths to any given ONU is $C = (\Delta_1 + \Delta_2 + \dots + \Delta_N)/(L - \rho)$.*

Proof: The result may be deduced for periodic polling from Section 3 in [3]. The following alternative proof is valid also for random polling. Since the system is stable, the mean number of wavelengths transmitting data is ρ . Thus $L - \rho$ is the mean number of wavelengths currently busy due to switch overhead. The fraction of time a server at ONU i experiences a switch overhead is Δ_i/C . The relation for C is obtained by summing up these quantities. ■

C. Server limits, no grant limits

The capacity of a multiserver polling system with server limits but unlimited gated service with periodic or random polling is given by the following.

Proposition 3. *When the WDM EPON with tunable transmitters has unlimited gated service, it is stable if and only if $\rho_i < t_i$ for $1 \leq i \leq N$ and $\rho_1 + \rho_2 + \dots + \rho_N < L$.*

Proof: The conditions are clearly necessary since otherwise at least one user queue would be unstable. To prove sufficiency, we use fluid limit arguments as in [6], [7]. Starting from any initial conditions, it is necessary to prove that the overall fluid content denoted $\bar{W}(t)$ decreases with time. Let $\mathcal{F}(t)$ designate the number of ONU queues that have a fluid backlog at t . If $\sum_{i \in \mathcal{F}(t)} t_i \geq L$, all the service capacity is devoted to fluid queues and $\bar{W}(t) = \sum_i \rho_i - L < 0$ by the second condition. If $\sum_{i \in \mathcal{F}(t)} t_i < L$, all queues in $\mathcal{F}(t)$ will be served at their maximum rate. We have, therefore, $\bar{W}(t) = \sum_{i \in \mathcal{F}(t)} (\rho_i - t_i) < 0$. ■

Combining server limits and grant limits leads to a system whose traffic capacity turns out to be intractable, even for the simplest non-trivial configuration of 3 ONUs served by 2 wavelengths. This motivates the large system approximations discussed in the next section.

IV. LARGE SYSTEM ASYMPTOTIC

We derive the traffic capacity of a WDM PON with tunable transmitters under the assumption that a mean field limit is valid in large systems.

A. Negligible switch overhead

As a preliminary, consider the polling system representing a WDM PON under the assumption that overhead (for switching between ONUs) is negligibly small. Assume the ONUs are each equipped with one tunable transmitter. Let the number of active ONUs at time t be $Z(t)$, i.e., $Z(t)$ is the number of

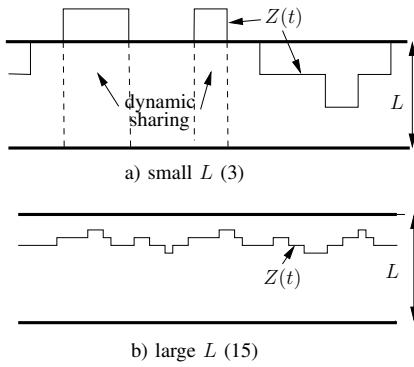


Fig. 2. Sharing L wavelengths between $Z(t)$ active queues - no switch times

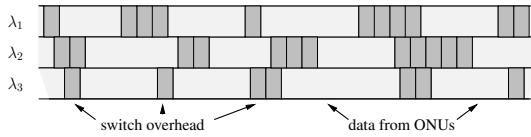


Fig. 3. Occupation of a 3-wavelength PON by data and overhead

ONUs with at least one backlogged queue. When $Z(t) \leq L$, every ONU is served by one wavelength. If $Z(t) > L$, on the other hand, the PON shares the capacity of the L wavelengths according to the DBA algorithm.

Figure 2 illustrates the importance of the value of L . In the top figure, $L = 3$ and, depending on the ONU loads, it is a frequent occurrence that $Z(t) > L$. ONUs then share capacity dynamically and it is not easy to predict PON performance. In the lower figure, $L = 15$ and the fluctuations of $Z(t)$ have relatively lower amplitude. In the depicted realization, every PON is always able to use its tunable transmitter whenever it is active.

As L and N increase indefinitely, it is intuitively clear that the fluctuations of $Z(t)$ become small compared to L . Each ONU then behaves like an independent polling system with the server capacity of one wavelength. Every ONU queue is stable if $\rho_i < 1$ for $1 \leq i \leq N$ and $\rho < L$.

This intuition is confirmed by the propositions of Section V in the case where ONUs have a single queue, service is limited to one packet per cycle, packet arrivals are Poisson and packet sizes exponential. A more general proof is, for the moment, out of reach. We nevertheless proceed in this and the next sections with the reasonable assumption that a ‘‘mean field’’ asymptotic is valid as L and N both tend to infinity.

It is clear that the above capacity limits can be generalized to ONUs equipped with a variable number of tunable transmitters. In a large system, each ONU always has access to its quota of wavelengths whenever it is active. The traffic capacity of ONU i is thus that of a t_i -server polling system: queues are stable if $\rho_i < t_i$ for $1 \leq i \leq N$ and $\rho < L$.

B. Limited service polling system with switch overhead

We now suppose switch overheads are non-negligible: each visit to ONU i consumes Δ_i seconds of wavelength capacity. Again, first assume each ONU is equipped with just one

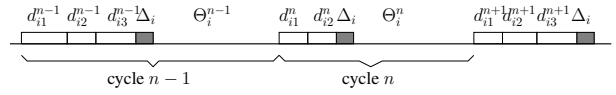


Fig. 4. ONU local polling system

tunable transmitter. Figure 3 shows how wavelengths are fully used in a WDM PON. Each wavelength is either transmitting data or is unavailable because of the switch overhead. When $\rho < L$, the visit frequency to inactive ONUs is such that residual capacity $L - \rho$ is entirely consumed by overhead.

Figure 4 illustrates the upstream activity of an ONU. This ONU has 3 queues that are served cyclically respecting maximum grant limits on each visit. In the figure, queue 3 happens to be empty in cycle n . Each set of grants terminates with the switch overhead. After the overhead, the wavelength visits another free ONU (possibly this one again) and the considered ONU sees a vacation time (≥ 0) before it is again visited by a wavelength.

When the system is stable, at equilibrium the vacation time of ONU i , i.e., the time when no server is present, is denoted by Θ_i . Note that this random variable depends *a priori* on the activity of all ONUs. Under the mean field assumption below, this dependence vanishes in the large system limit where N and L tend to infinity. Let $\theta = \mathbb{E}[\Theta_i]$ and δ be the mean switchover time at equilibrium over all ONUs. First, we derive a useful general relation.

Proposition 4. *At equilibrium, the mean vacation interval θ of a system with one tunable transmitter per ONU satisfies*

$$\theta = \frac{(N - L)\delta}{L - \rho}. \quad (3)$$

Proof: At equilibrium, the number of ONUs that are not currently being served is constant and equal to $N - L$. Consequently $(N - L)/\theta$ is the rate at which one of these ONUs receives the visit of a server. Similarly, the expected number of wavelengths transmitting data must be equal to the offered traffic ρ . The remainder, $L - \rho$, is the mean number of servers in an overhead period and consequently $(L - \rho)/\delta$ is the rate at which servers leave a switch overhead period. The identity of the proposition follows. ■

To exploit Equation (3) we must make the following additional assumption.

Mean field assumption. We assume successive intervals Θ_i^n are independent and identically distributed for all i and all n with mean θ . This means the local ONU polling system with vacation depicted in Figure 4 can be analyzed as an independent system, the impact of the other ONUs being manifested through the value of the mean field parameter θ .

We maintain this assumption is reasonable when N and L are large and when, for each wavelength, the choice of the next ONU to visit is made uniformly at random among those that are eligible. The latter assumption applies for both random polling and periodic polling in the large system limit. This, roughly speaking, is because the past activity of any ONU has negligible impact on future wavelength visits while available

destinations for a wavelength's next visit hardly depend on its own history. The system is effectively memoryless and, indeed, we would further claim the distribution of the Θ_i^n is exponential in the large system limit. This property is not used for the present capacity results.

To evaluate θ , note that, under the mean field assumption, the system represented in Figure 4 behaves like a classical single server periodic polling system with limited gated service and expected overhead per cycle equal to $\Delta_i + \theta$. By Proposition 1, its mean cycle time is $(\Delta_i + \theta)/(1 - \rho_i)$. Straightforward calculations show that the overall mean overhead δ defined above can then be expressed as

$$\delta = \frac{\sum(1 - \rho_i)\Delta_i/(\Delta_i + \theta)}{\sum(1 - \rho_i)/(\Delta_i + \theta)}. \quad (4)$$

Eliminating δ in (3) and (4) yields an equation that can be solved for θ . When the Δ_i are all equal (to δ), θ is given directly by (3). Otherwise, we must resort to numerical evaluation.

Stability conditions result from those of the individual ONU polling systems where the overhead and vacation are assimilated to the per-cycle switch time.

Proposition 5. *Under the mean field assumption, the WDM PON with one tunable transmitter per ONU, represented as a multiserver, limited-gated polling system where no more than one server can simultaneously attend any queue, is stable if and only if*

$$\rho_i + \frac{\rho_{ij}}{d_{ij}}(\Delta_i + \theta) < 1, \quad 1 \leq i \leq N, \quad 1 \leq j \leq n_i. \quad (5)$$

where θ is derived from (3) and (4).

The proof follows directly from Proposition 1 on making the appropriate substitutions. When the overhead is the same for each ONU we readily deduce the following.

Corollary 1. *When $\Delta_i = \delta$ for all i , every queue is stable if and only if*

$$\rho + \max_{ij} \left\{ \frac{\rho_{ij}}{d_{ij}} \frac{\delta(N - \rho)}{1 - \rho_i} \right\} < L. \quad (6)$$

C. Local stability

Limited gated service is designed to limit the impact on ONU performance when some users saturate their share of PON capacity. Let the set of saturated queues be \mathcal{S} . To derive stability conditions for the other queues we must simply replace Δ_i by $\Delta_i + \sum_{j:(i,j) \in \mathcal{S}} d_{ij}$ in (4) to compute θ and substitute for Δ_i and θ in (5).

For a system with given traffic distribution, the following algorithm determines the set \mathcal{S} of saturated queues.

- 1) $\mathcal{S} = \phi$, the empty set.
- 2) find $(i, j) = \operatorname{argmax}_{(i,j) \notin \mathcal{S}} \left\{ \frac{\rho_{ij}}{d_{ij}} \Delta_i \right\}$.
- 3) if (5) is satisfied for (i, j) then stop.
- 4) else,
 - $\Delta_i \leftarrow \Delta_i + d_{ij}$,
 - $\rho \leftarrow \rho - \rho_{ij}$,

— $\mathcal{S} \leftarrow \mathcal{S} \cup (i, j)$.

5) repeat from step 2.

D. Multiple transmitters

To simplify the presentation, we have so far assumed each ONU has exactly one tunable transmitter. However, the above results apply immediately with slight modification to a variable number of transmitters if we make the following assumption. The wavelengths on terminating one service visit an eligible ONU with probability proportional to its number of free transmitters. This would result in the large system limit if each wavelength had a fixed cycle of visits chosen as a random permutation of *transmitters*. In condition (5), it is necessary to replace 1 on the right hand side by t_i .

E. Frame based GPON DBA

In GPON, a possible DBA is to divide the capacity available in a frame (or several successive frames) between ONU queues, applying a max-min fair algorithm. The overhead per frame is fixed and equal to $\sum \Delta_i$. The residual capacity would be shared by the OLT between active ONU queues. Queue j of ONU i would receive an allocation when active proportional to some weight w_{ij} .

Assume the frame duration f and the Δ_i tend to zero while the ratio Δ_i/f remains equal to δ_i . The underlying service system is now weighted head of line processor sharing with a maximum service rate for ONU i equivalent to $t_i(1 - \delta_i)$.

The capacity of this system appears intractable in general [4]. In the large system limit where the number of queues and the system capacity increase indefinitely, however, we intuitively expect the service rate constraint to apply to all ONUs, i.e., with high probability, every ONU is served when busy at its maximum rate. The system in fact behaves as discussed in Section IV-A and the stability conditions are

$$\rho < L(1 - \sum \delta_i), \text{ and } \rho_i < t_i(1 - \delta_i). \quad (7)$$

V. PROOF OF MEAN FIELD CONVERGENCE

The above results rely on the assumption that, as N and L get large, the individual ONU queue sets behave as independent systems. This is a classical mean field assumption that would ideally need formal justification. In this section we prove the mean field property under some further simplifying assumptions.

Homogeneous limited server polling system without switch overhead times. We consider an S_N -server polling system where each of N queues can be served by only one server at a time. After each service, a server chooses a new, non-empty queue uniformly at random. Moves between queues are instantaneous, there is no switchover time. Customer arrivals are Poisson of intensity λ in each queue and service times are exponential with rate μ . The queue load is $\rho = \lambda/\mu$.

For $t \geq 0$, let $Q_i(t)$ be the number of customers in the i^{th} queue at time t and $I_i(t) \in \{0, 1\}$ an indicator of the presence of a server at queue i . Clearly, if $Q_i(t) = 0$ then $I_i(t) = 0$. The process

$$(X(t)) \stackrel{\text{def.}}{=} (Q_n(t), I_n(t), 1 \leq n \leq N)$$

is then a Markov process taking values in $\mathcal{S} = \{x = (x_n, i_n) \in (\mathbb{N} \times \{0, 1\})^N : x_n > 0 \text{ if } i_n = 1\}$.

We consider a ‘‘large’’ system where

$$\lim_{N \rightarrow +\infty} \frac{S_N}{N} = s, \text{ with } s \in (0, 1).$$

Note that the ‘‘natural’’ necessary stability conditions are $N\rho < S_N$ or, in the limit, $\rho < s$.

First assume the initial occupancies $Q_i(t)$ of each queue are independent and geometrically distributed with parameter ρ . As N grows large, we have

$$\sum_{n=1}^N \mathbf{1}_{\{Q_n(0) > 0\}} \sim N\mathbb{P}(Q_1(0) > 0) = N\rho < Ns \sim S_N.$$

Under this regime there is an infinite number of idle servers so that the vector $(I_n(0))$ is uniquely determined. The following proposition shows that this holds with high probability on any finite time interval.

Proposition 6. *If $\rho < s$ and the initial state $(X(0)) = ((Q_n(0), I_n(0)))$ is such that the variables $Q_n(0)$ are independent with a geometric distribution with parameter ρ then, for each $T \geq 0$, there exists $\varepsilon > 0$ such that*

$$\lim_{N \rightarrow +\infty} \mathbb{P} \left(\sup_{0 \leq s \leq T} \sum_{n=1}^N \mathbf{1}_{\{Q_n(t) > 0\}} < S_N - \varepsilon N \right) = 1.$$

Proof: A simple coupling argument is used. Consider N independent $M/M/1$ queues with lengths $L_n(t)$ with arrival rate λ and service rate μ at equilibrium. For each $t \geq 0$, $L_n(t)$, $1 \leq i \leq N$, are then N independent geometric random variables with parameter ρ . Let

$$A_N(t) = \frac{1}{N} \sum_{n=1}^N \mathbf{1}_{\{L_n(t) > 0\}}.$$

By the law of large numbers, almost surely,

$$\lim_{N \rightarrow +\infty} A_N(t) = \rho.$$

In order to prove a stronger statement, i.e., that this convergence holds not only for a fixed time but on a whole time interval, one has to work in the Skorokhod space of probability distributions on real valued functions on $[0, T]$ with limits on the left and continuous on the right. See Billingsley [2], for example.

In the following, $(\mathcal{N}_{x,i})$ denotes an i.i.d. sequence of Poisson processes with parameter x and $\mathcal{N}_{x,1}([a, b])$ is the number of points in the interval $[a, b]$. We first prove that the sequence of processes $(A_N(t))$ is tight. Since a large variation of $(A_N(t))$ can only be due to either arrivals or departures,

we have, for δ and $\eta > 0$,

$$\begin{aligned} & \mathbb{P} \left(\sup_{1 \leq s, t \leq T: |t-s| \leq \delta} |A_N(t) - A_N(s)| \geq \eta \right) \\ & \leq \mathbb{P} \left(\sup_{1 \leq s, t \leq T: |t-s| \leq \delta} \frac{1}{N} \sum_{n=1}^N \mathcal{N}_{\lambda, n}([s, t]) \geq \frac{\eta}{2} \right) \\ & \quad + \mathbb{P} \left(\sup_{1 \leq s, t \leq T: |t-s| \leq \delta} \frac{1}{N} \sum_{n=1}^N \mathcal{N}_{\mu, n}([s, t]) \geq \frac{\eta}{2} \right). \end{aligned}$$

Using the fact that $\mathcal{N}_{\lambda, 1} + \dots + \mathcal{N}_{\lambda, N}$ has the same distribution as $\mathcal{N}_{N\lambda, 1}$, by the law of large numbers for Poisson processes, the first term of the right hand side of the above expression becomes arbitrarily small as N grows if δ is less than $\eta/2\lambda$. Similarly, the second term has a similar property. We deduce from this estimate that the sequence of processes $(A_N(t), 0 \leq t \leq T)$ is tight for the convergence of the uniform norm and, therefore, that any of its limiting points is a continuous process. Since the marginals converge to the constant ρ , the only limiting process is the constant process equal to ρ .

The sequence $(A_N(t), 0 \leq t \leq T)$ thus converges as a process to (ρ) . In particular, the supremum of $(A_N(t))$ on the time interval $[0, T]$ converges to ρ . Since $S_N \sim sN > \rho N$, for $\varepsilon < s - \rho$, we have

$$\lim_{N \rightarrow +\infty} \mathbb{P} \left(\sup_{0 \leq t \leq T} A_N(t) < \frac{S_N}{N} - \varepsilon \right) = 1.$$

On the event $\{\sup_{0 \leq t \leq T} A_N(t) < S_N/N - \varepsilon\}$, there is always an idle server. In this situation, polling system $(X(t))$ is equivalent to N independent $M/M/1$ queues so that $(L_n(t), 0 \leq t \leq T)$ has the same distribution as $(Q_n(t), 0 \leq t \leq T)$. ■

The following corollary states the required mean field property given the assumed initial occupancy distribution.

Corollary 2. *Under the assumptions of the above proposition, on any finite time interval, as N gets large, the Markov process $(X(t))$ has the same distribution as N independent $M/M/1$ queues. In this limit, the natural condition $\rho < s$ is the stability condition of the system.*

We conclude this section by a result which states that, starting from any initial state, the process very rapidly enters a set of states where many servers are idle.

Proposition 7. *If $\rho < s$ and $x = (q_n, i_n) \in \mathcal{S}$ is such that $|\{n : q_n > 0\}| > S_N$ and $q_1 + \dots + q_N < CN$ for some constant C , the infimum*

$$T_N = \inf\{s, |\{n, Q_n(t) > 0\}| < S_N\},$$

satisfies

$$\frac{1}{N} \mathbb{E}_x(T_N) \leq \frac{C}{\mu S_N/N - \lambda}.$$

Proof: Define, for $t \geq 0$,

$$Z(t) = Q_1(t) + \dots + Q_N(t).$$

Since all servers are busy at time 0 by assumption, as long as $t < T_N$, $Z(t) - Z(0)$ can be represented as the difference of two Poisson processes with respective parameters λN and μS_N , so that $(Z(t \wedge T_N) + (\mu S_N - N\lambda)(t \wedge T_N))$ is a supermartingale. In particular

$$\mathbb{E}(Z(t \wedge T_N) + (\mu S_N - N\lambda)(t \wedge T_N)) \leq Z(0) \leq CN.$$

Hence, $(\mu S_N - N\lambda)\mathbb{E}(t \wedge T_N) \leq CN$ and the desired inequality follows on letting t go to infinity. ■

VI. EVALUATION

We report results of a preliminary evaluation of the accuracy of the mean field approximation and quantify the impact of the switch overhead.

A. Convergence to mean field

In the simulations, we consider a realistic EPON configuration where each wavelength provides an upstream transmission bit rate of 1 Gbps. We assume that ONUs have a single queue with infinite capacity and one tunable transmitter. Users send fixed size 1000 byte packets according to a Poisson process. The grant size is fixed to $8 \mu s$ (transmission time of one packet) and switch overhead is set to $1.2 \mu s$ (15% of the grant size).

Figure 5 shows the proportion of wavelengths in switch overhead measured in 1ms intervals for $N = 10, 50, 100$ and 500, respectively. Overall load is $0.2N$ and $L/N = 0.5$. The figure shows that, as the number of ONUs increases, the fluctuations of this proportion decrease. The wavelength visit rate to a given ONU, determined by this proportion, thus converges to a constant. This is why it is plausible that individual ONUs tend to become statistically independent.

B. Capacity region

Figure 6 depicts the capacity region of an EPON with two classes of ONUs. Each ONU of the same class has the same load and there is an equal number of ONUs per class. $L/N = 1/2$. The region to the left and below the plots corresponds to loads for which the ONU queues are stable. Units are normalized loads with respect to the total number of ONUs.

Figure 6 compares the capacity region determined by (6) with simulation results for $N = 6$ and $N = 20$. The accuracy of the results is good for $N = 6$ and excellent for $N = 20$. Discrepancies for $N = 6$ with unbalanced traffic are not surprising as the number of ONUs is then effectively only 3. Similar precision is observed in Figure 7 where class 2 has three times as many ONUs as class 1. Note that PONs are currently designed for around 100 ONUs so that the accuracy of the mean field approximation is largely sufficient.

C. Impact of overhead

Figure 8 illustrates the impact of the overhead as a percentage of the grant limit given by the mean field approximation. For these results, $L/N = 0.4$ and 40% of ONUs are in class 1. The graphs show that the impact of overhead is

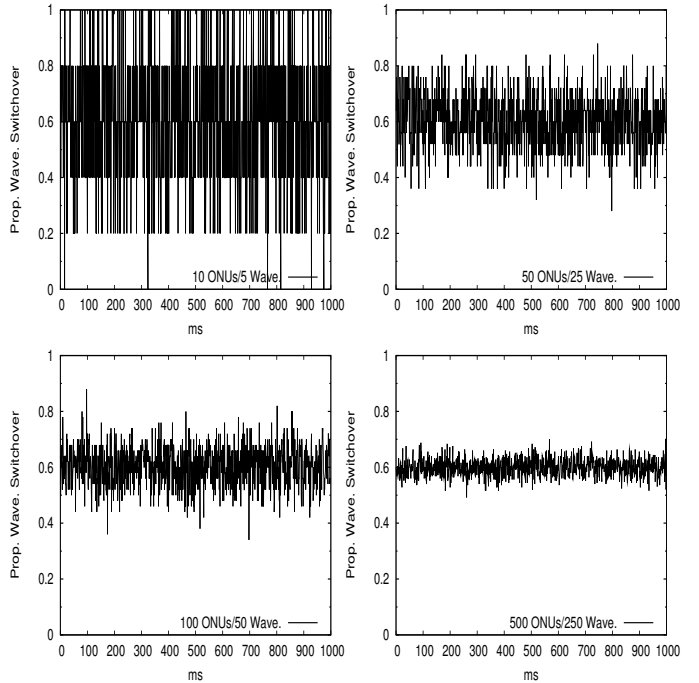


Fig. 5. Proportion of wavelengths in switch overhead in intervals of 1 ms for different numbers of ONUs ($N = 10, 50, 100$ and 500).

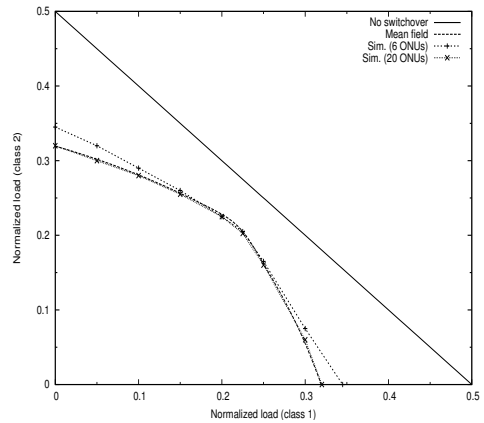


Fig. 6. Stability region for two classes of ONUs with balanced number of ONUs (curves for 20 ONUs and the mean field coincide; they define the smallest stability region).

significant. It is worthwhile increasing the grant size as long as the worst case latency remains sufficiently small. Note that when some queues are overloaded, their grant is effectively added to the overhead seen by the other queues, as discussed in Section IV-C. A 10% overhead might then not be unrealistic.

VII. CONCLUSION

The design of dynamic bandwidth allocation algorithms for WDM PONs brings challenging performance problems, even when we suppose the DBA does not attempt to realize intra-ONU service differentiation. The main performance criterion for these systems is the maximal traffic capacity they can

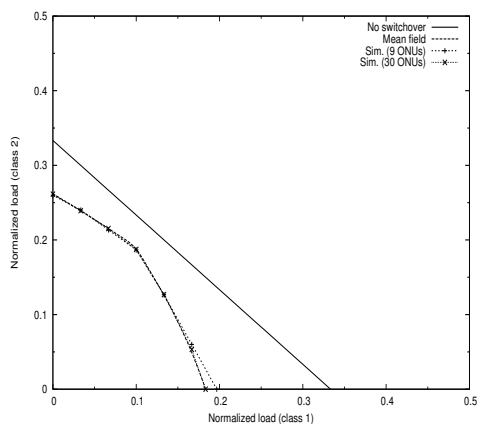


Fig. 7. Stability region for two classes of ONUs with unbalanced number of ONUs (curves for 30 ONUs and the mean field approximation coincide; they define the smallest stability region).

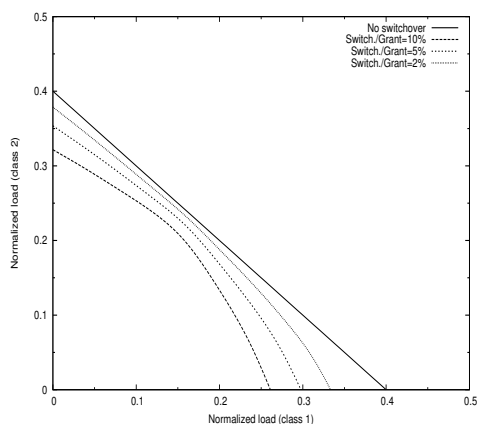


Fig. 8. Stability region for two classes of ONUs with unbalanced number of ONUs for different ratios switchover/Grant Size.

achieve, delays being negligibly small until loads are very close to this capacity.

The natural stochastic models to be investigated in this domain are multiserver polling models with both server limits and grant limits. They appear to be not tractable in general. For this reason, we have explored a large system approximation based on a mean field assumption. Under this assumption, the behaviour of a given ONU can be described as an isolated queueing system independent of the other ONUs. The traffic capacity analysis for this limiting representation of the PON notably reveals the non-trivial impact of switch overhead. Simulations reveal excellent accuracy even for systems much smaller than that of real PONs.

The mathematical justification of the mean field approximation has been provided in a simple case where switch times are null and traffic is Markovian. In future work we aim to extend this analysis to the realistic case of general traffic and non-zero switch overhead. We also intend to use the results derived here to perform a more thorough analysis of DBA performance for EPON and GPON.

Note finally that the derived large system approximations can be applied in other contexts where the natural model is a multiserver polling system with both server and grant limits.

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